

Redesigning the Economy to Achieve Carbon Transition

**THE SHIFT**  
PROJECT

# Growth engines and productivity Production functions

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# Why a production function for GDP ?

GDP cannot be decided politically

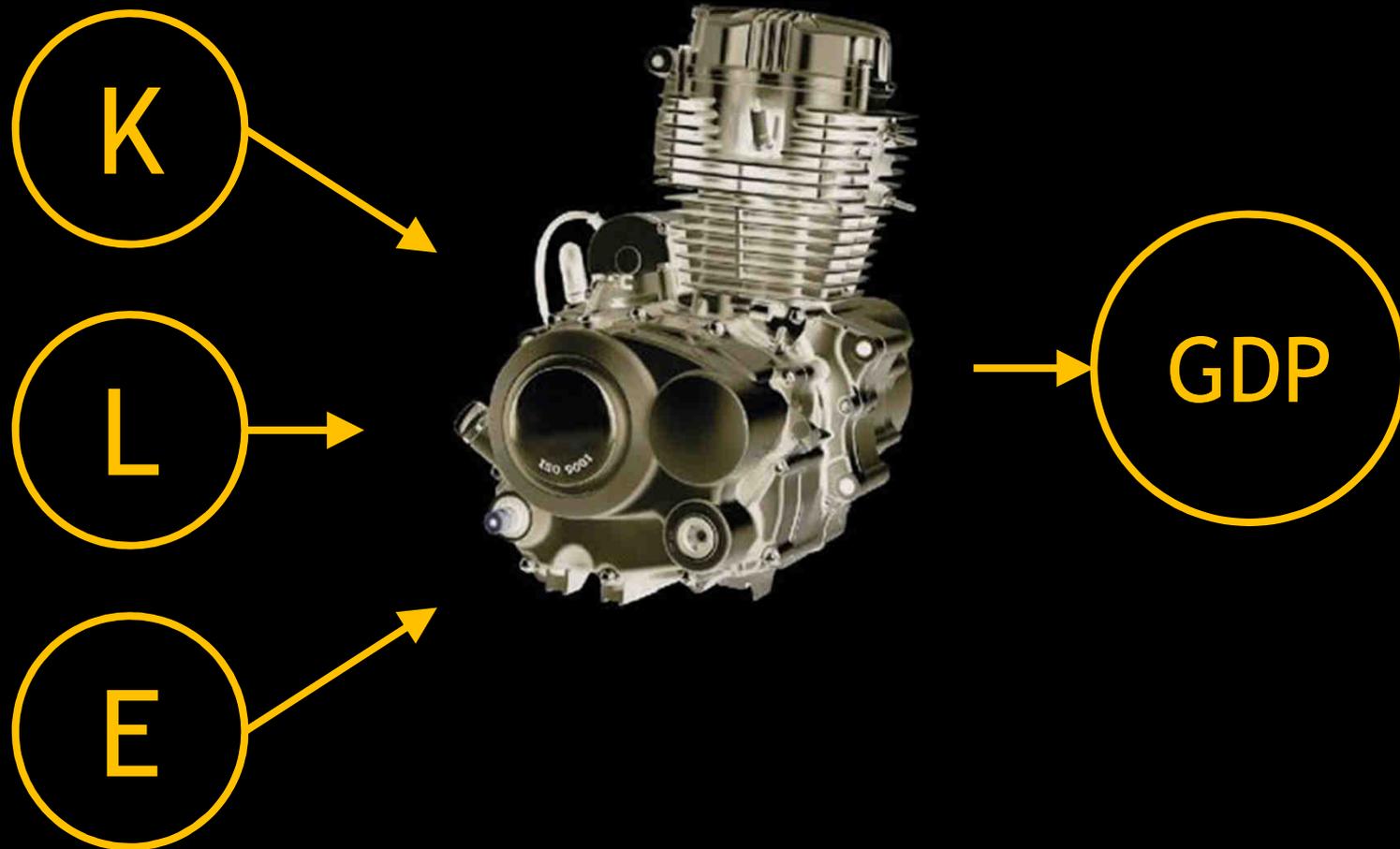
It depends on production factors

=> It **cannot** be an exogenous variable

# Some production functions

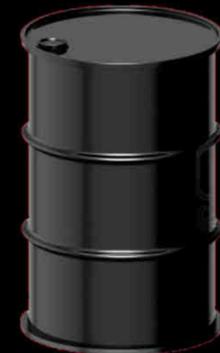
- Solow-Swan :  $Y = Y (K,AL)$
- + energy  $Y = Y (K,L,E)$
- ...or useful work :  $Y = Y (K,L,U)$
- + matter :  $Y = Y (K,L,E,M)$
- + creativity :  $Y = Y (K,L,E,C)$

# KLE production function



# Cobb-Duglas Function

$$Y = K^{\alpha} L^{\beta} E^{\gamma}$$



# Mathematical notation

$$Y = X_1^{\varepsilon_1} X_2^{\varepsilon_2} X_3^{\varepsilon_3}$$

$$X = (X_1, X_2, X_3)$$



3D space

## Each factor is essential

$$Y(0, X_2, X_3) = 0$$

$$Y(X_1, 0, X_3) = 0$$

$$Y(X_1, X_2, 0) = 0$$

Each factor is essential

$$Y = K^{\alpha} L^{\beta} E^{\gamma}$$



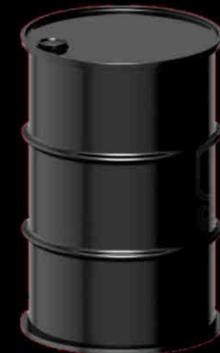
Each factor is essential

$$Y = 0^\alpha L^\beta E^\gamma = 0$$



Each factor is essential

$$Y = K^\alpha L^\beta E^\gamma = 0$$



Each factor is essential

$$Y = K^{\alpha} L^{\beta} 0^{\gamma} = 0$$



A production factor, not a destruction factor!

$$\frac{\partial Y}{\partial X_i} > 0$$

# Law of diminishing returns

$$\frac{\partial^2 Y}{\partial X_i^2} < 0$$

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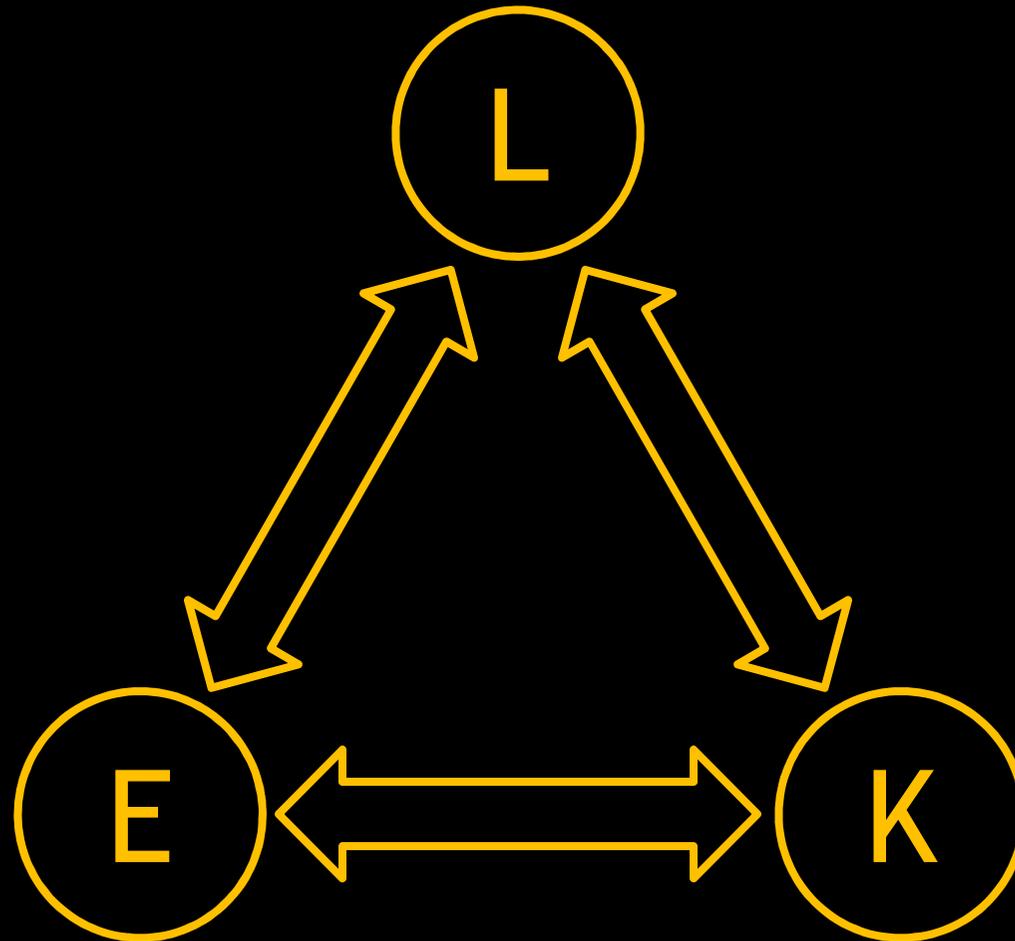
# Constant return to scale

$$Y(\lambda X) = \lambda Y(X)$$

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1$$



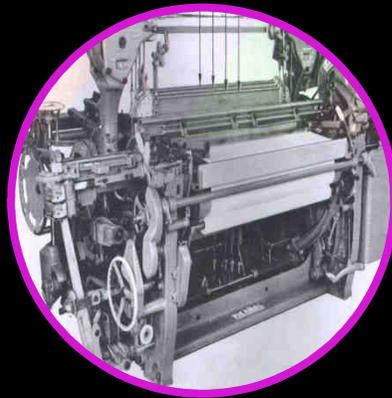
# Substitutability between factors



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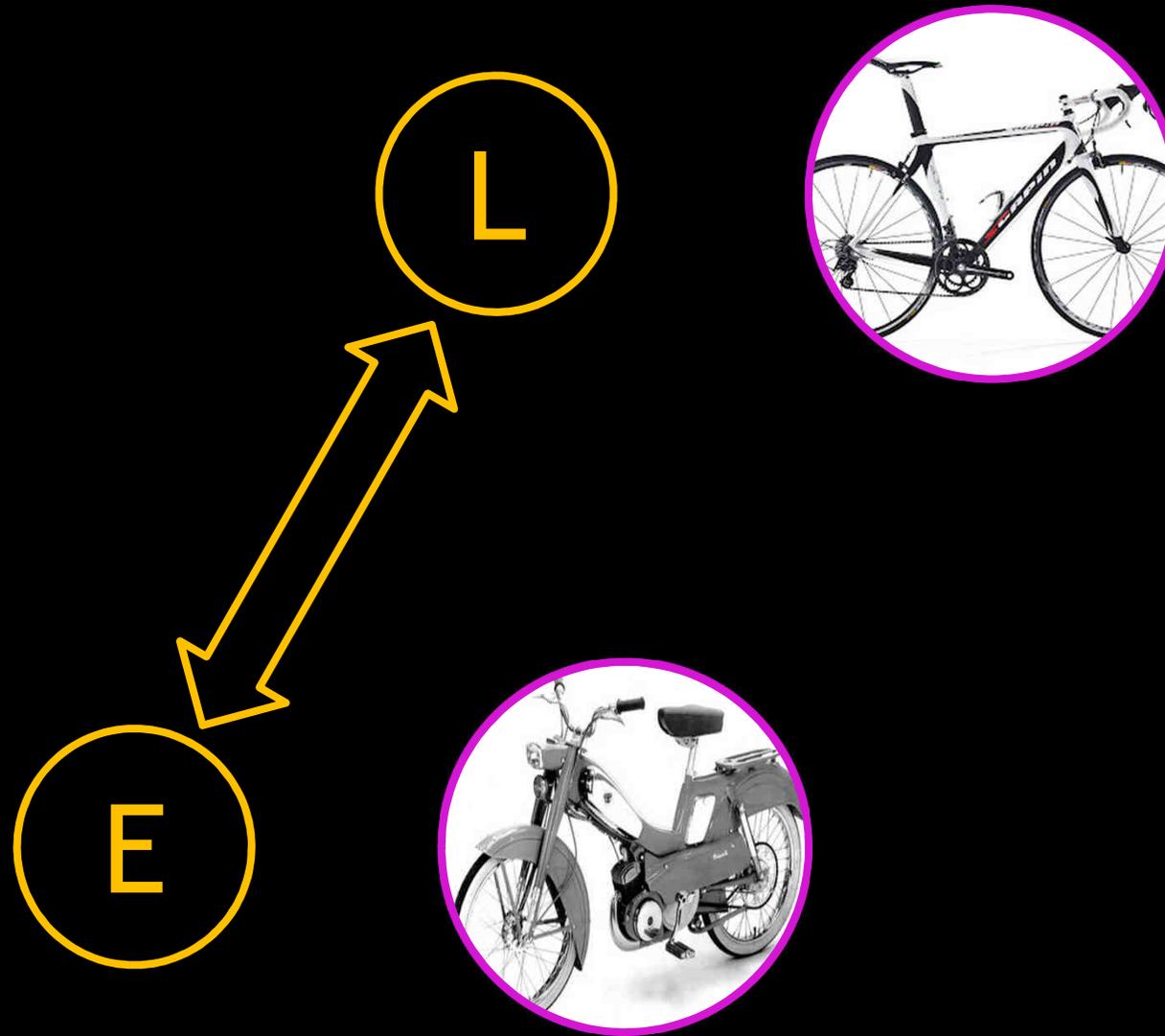
L



K



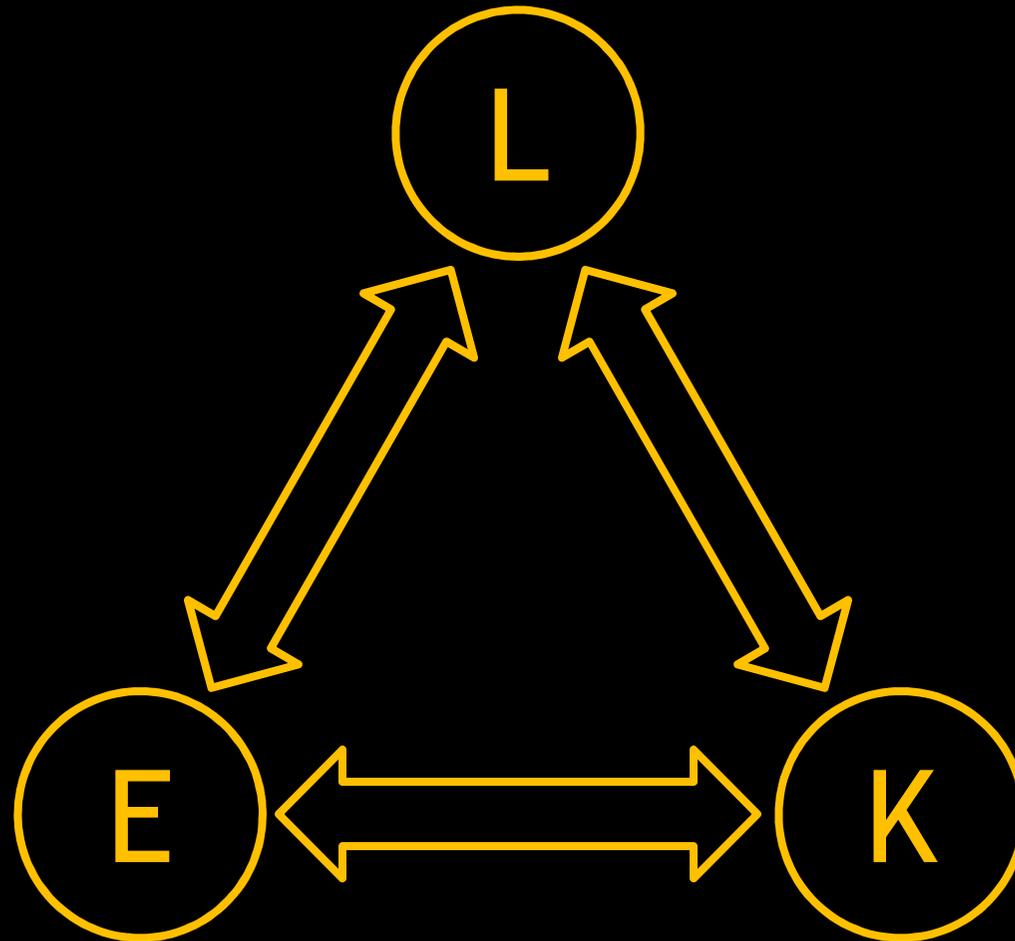
# Substitutability between factors



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## Substitutability has limits !

$$Y(0, X_2, X_3) = 0$$

$$Y(X_1, 0, X_3) = 0$$

$$Y(X_1, X_2, 0) = 0$$

But it is meaningful in the neighbourhood  
of today's point

# Logarithmic derivative

$$Y = X_1^{\varepsilon_1} X_2^{\varepsilon_2} X_3^{\varepsilon_3}$$

$$\frac{dY}{Y} = \varepsilon_1 \frac{dX_1}{X_1} + \varepsilon_2 \frac{dX_2}{X_2} + \varepsilon_3 \frac{dX_3}{X_3}$$

Output elasticity

# Output elasticity

$$\varepsilon_i = \frac{X_i}{Y} \frac{\partial Y}{\partial X_i}$$

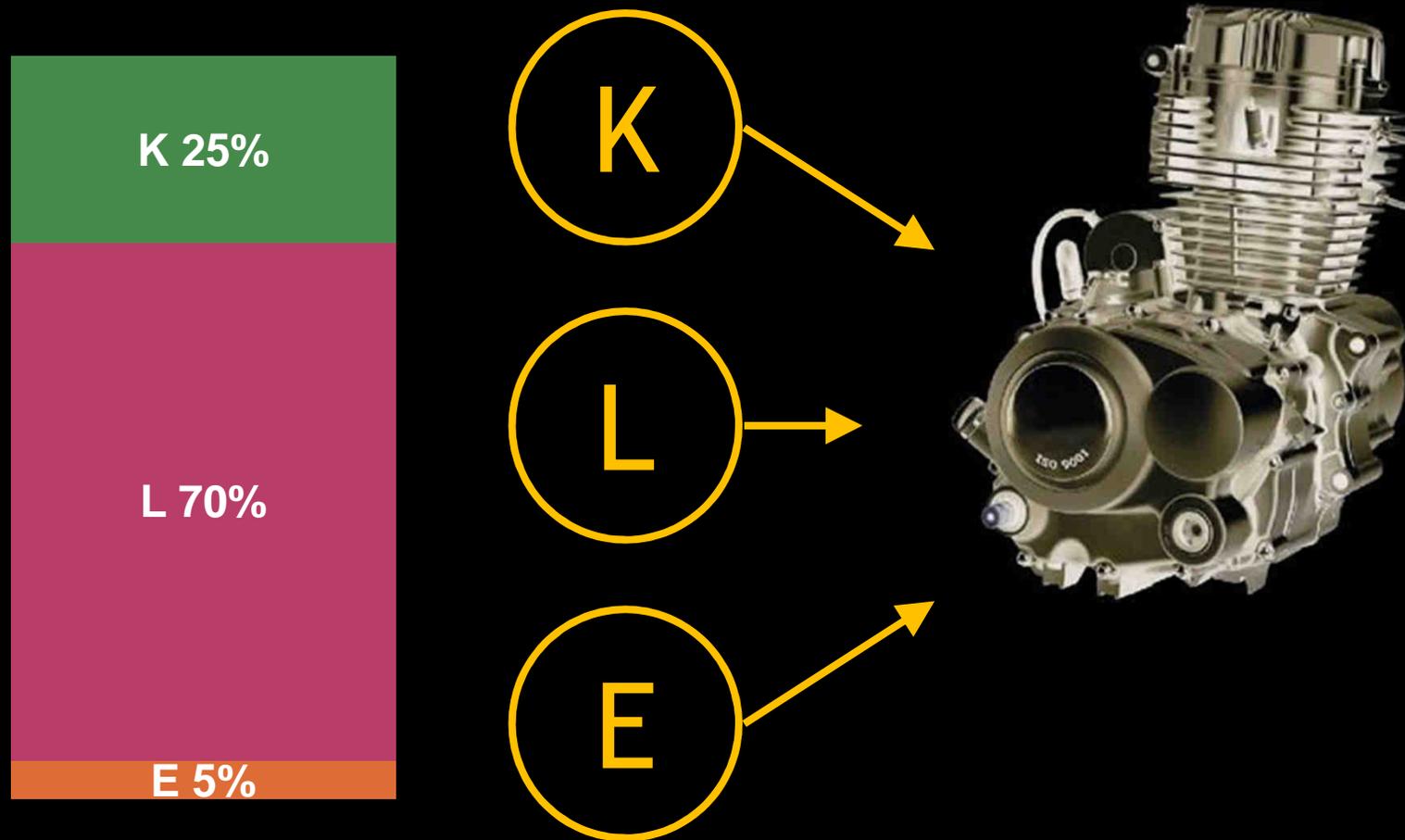
$$\varepsilon_i = \frac{\frac{\partial Y}{Y}}{\frac{\partial X_i}{X_i}}$$

# Cost Share Theorem

« At the equilibrium, the output elasticity of a production factor is equal to its cost share »

# Factor costs

Cost

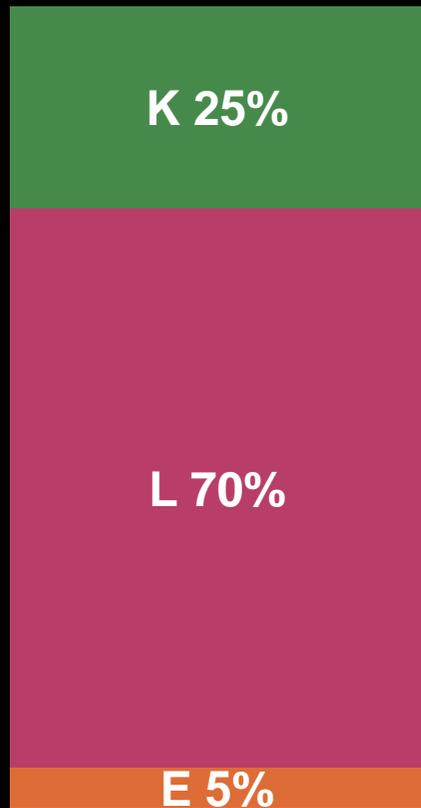


# Calculation of elasticity

- $Y < \text{GDP}$
- $K < \text{National accounting}$
- $L < \text{Ministry of Labour (Nb work hours)}$
- $E < \text{IEA (Primary energy consumption)}$

# Cost and elasticity

Cost

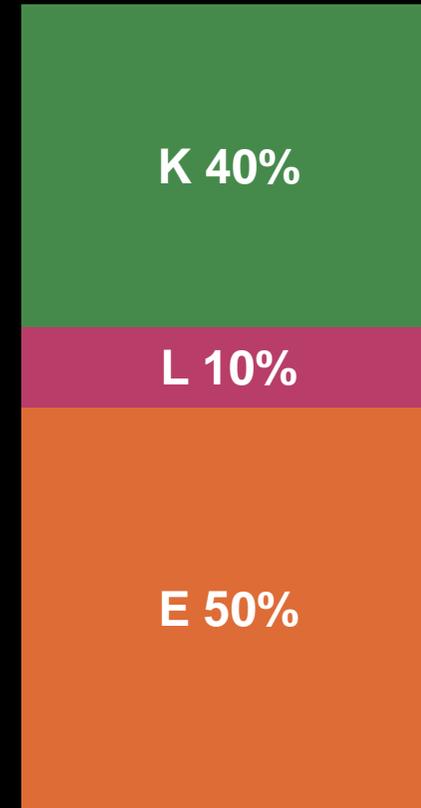


K

L

E

Elasticity



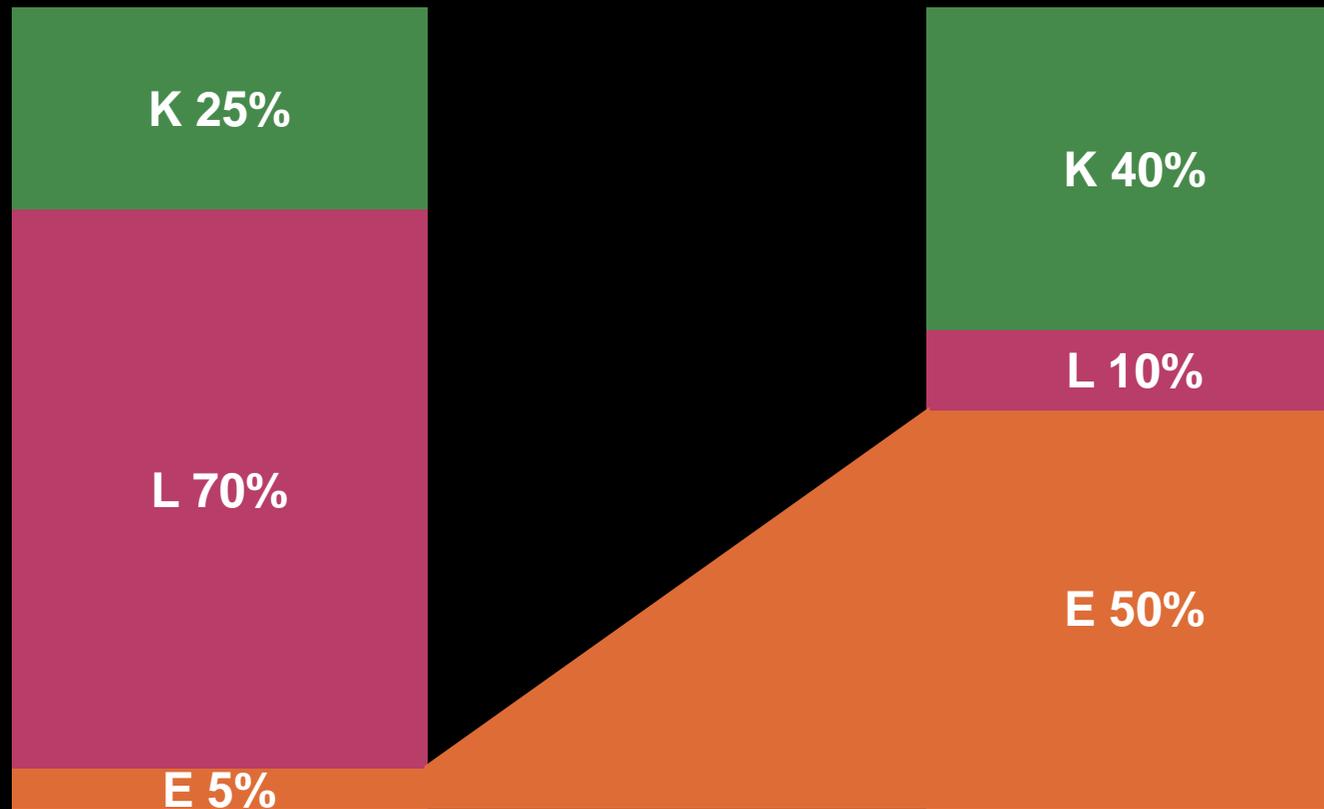
Ok, Reiner, so what do we do with that ?



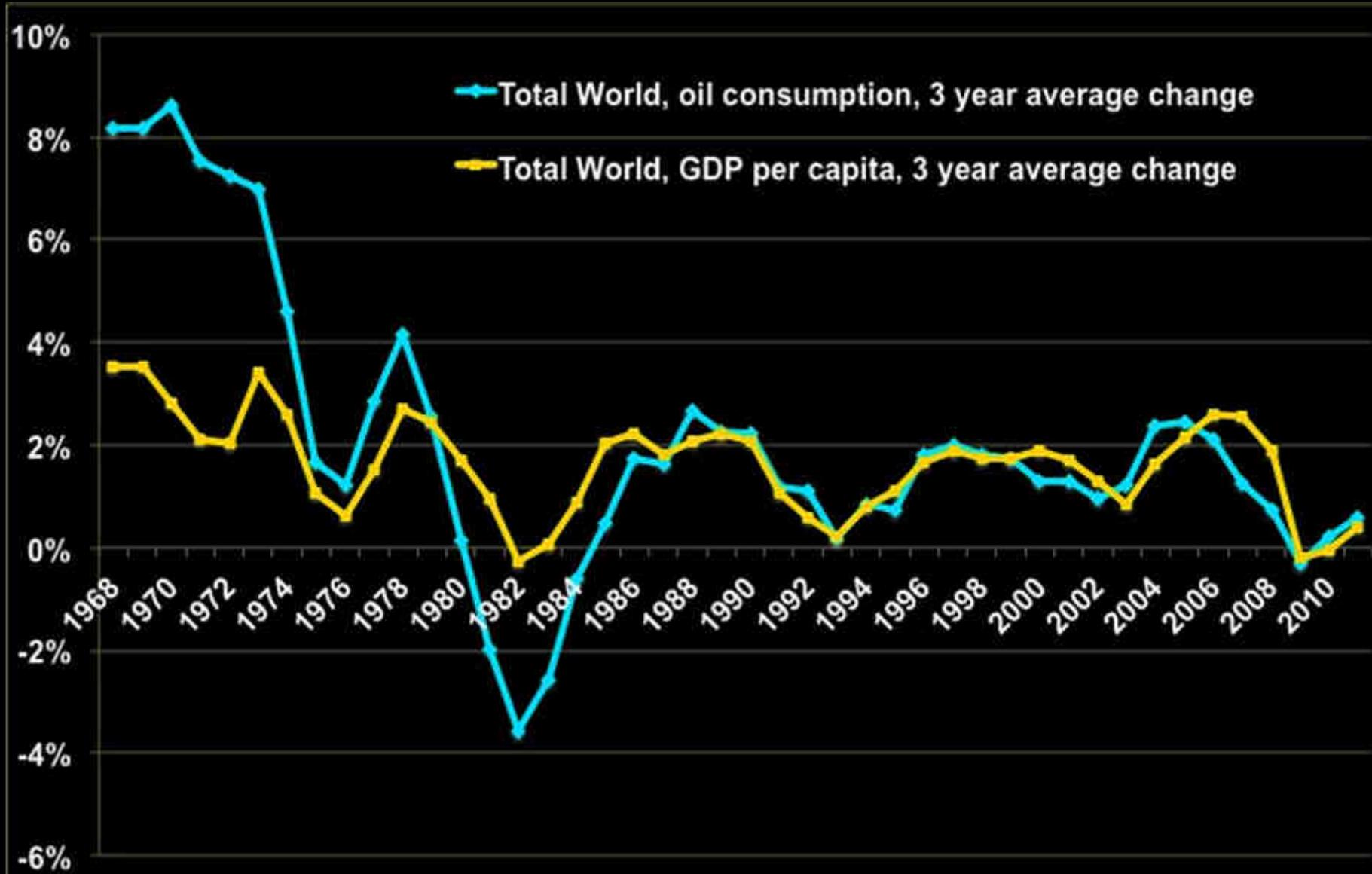
# 1) Energy is undervalued

Cost

Elasticity

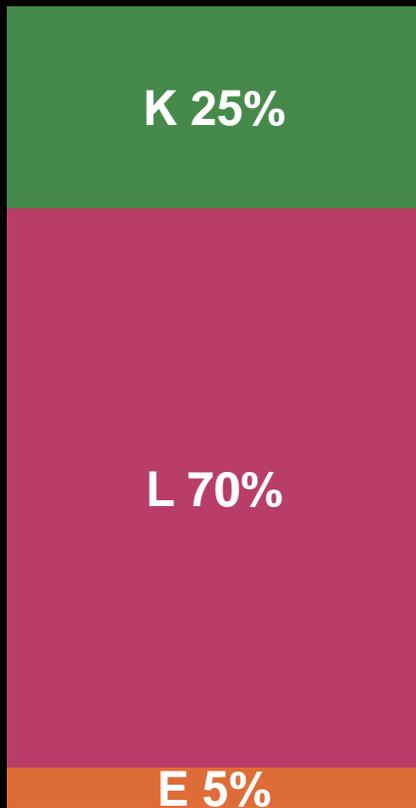


# Link between oil and economy

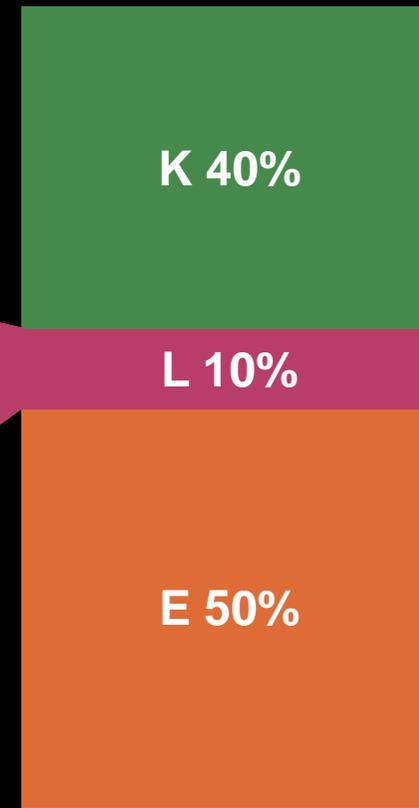


## 2) Labour is overpaid !

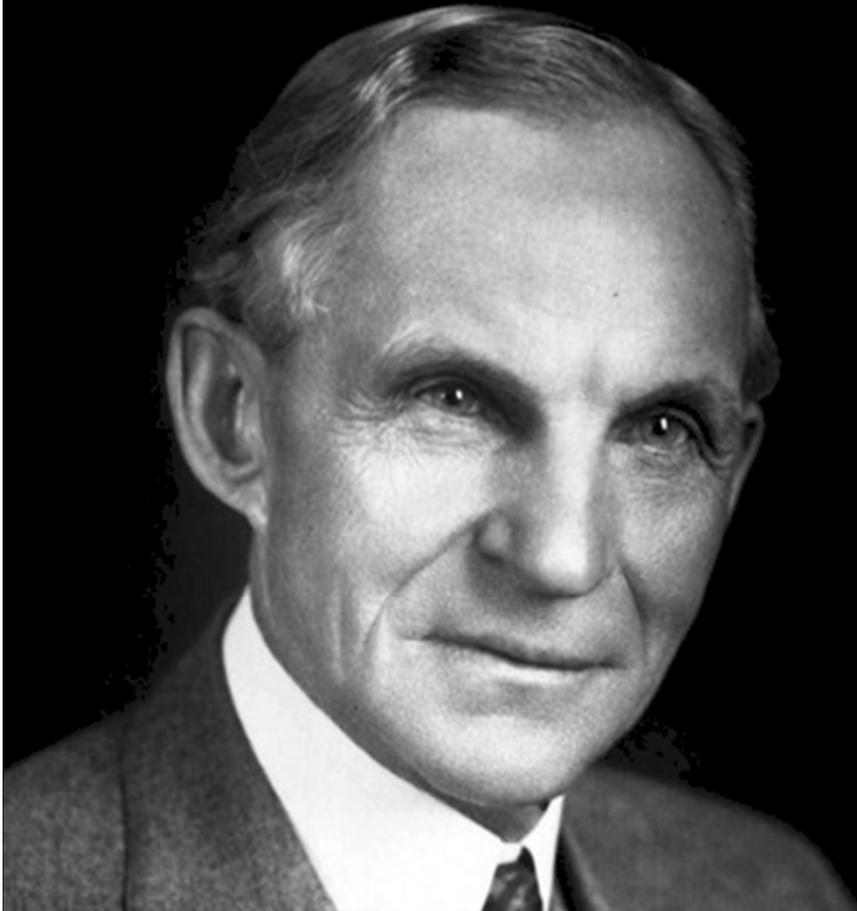
Cost



Elasticity



That's on purpose !

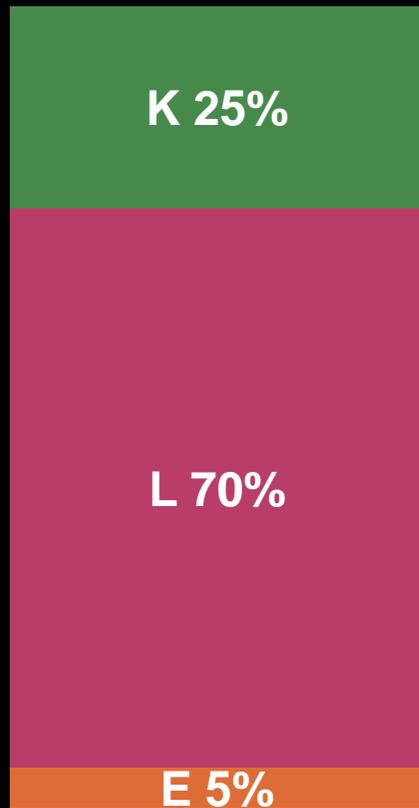


### 3) Energy cannot be substituted by Labour



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Cost

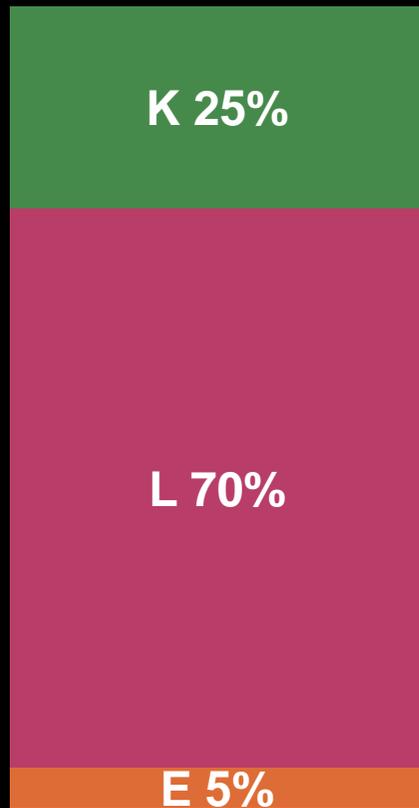


Elasticity

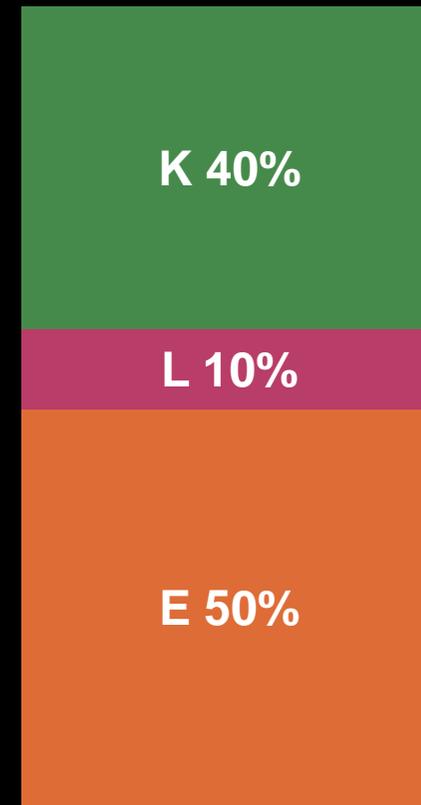


## 4) The solution : investment !!

Cost



Elasticity



## 4) The solution : investment !!





Stock

Flow



E

K

(Flow)

(Stock)

“We are moving from a OpEx to a  
CapEx world”

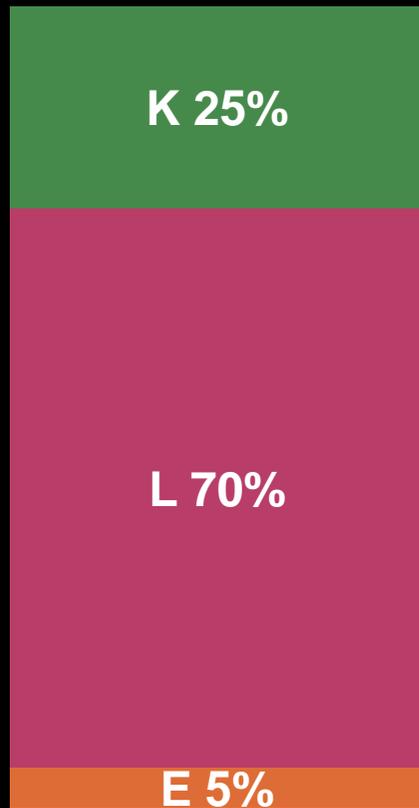
HSBC

Investment will be everywhere...



# 5) Question about capital remuneration

Cost



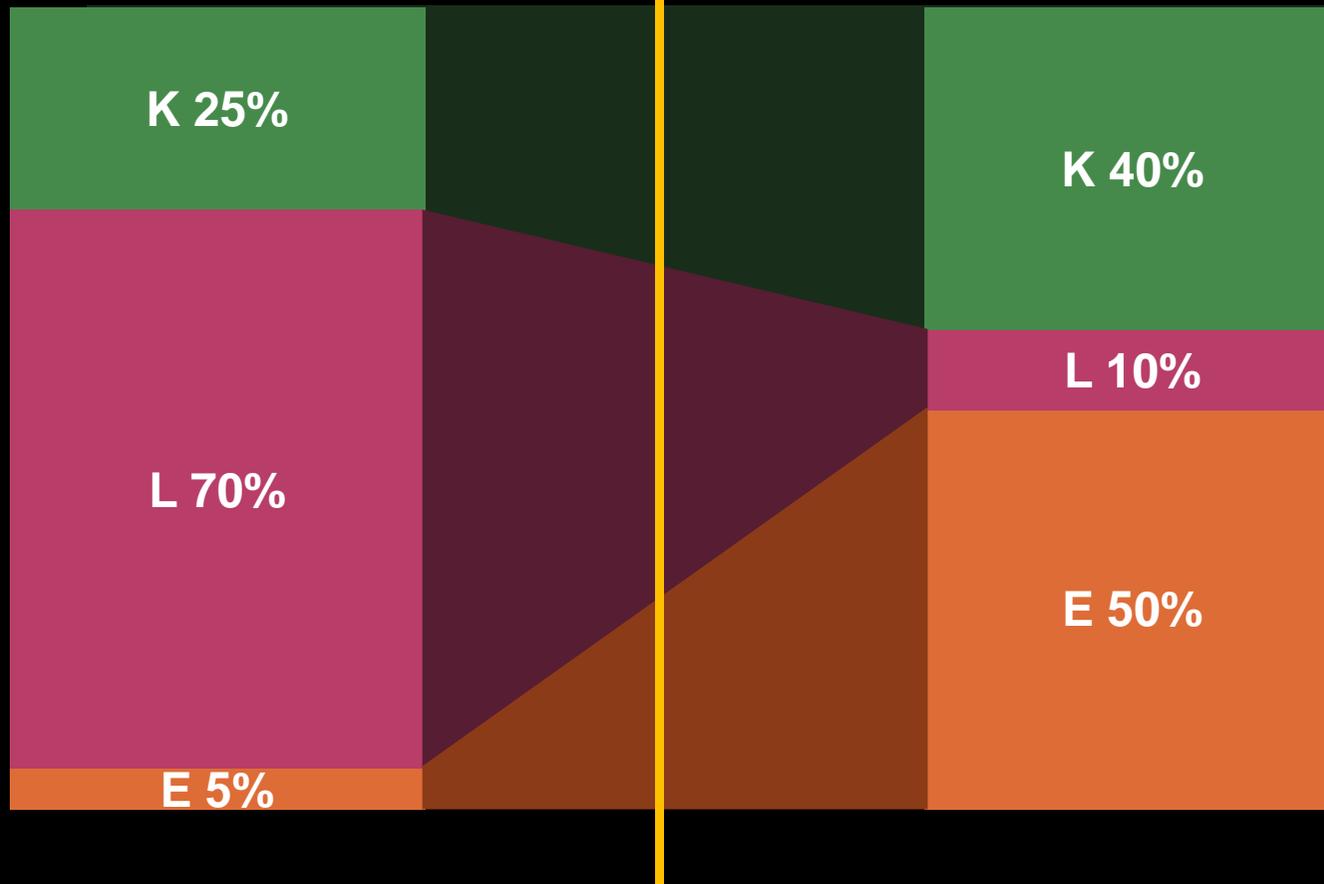
Elasticity



# Toward the equilibrium ?

Cost

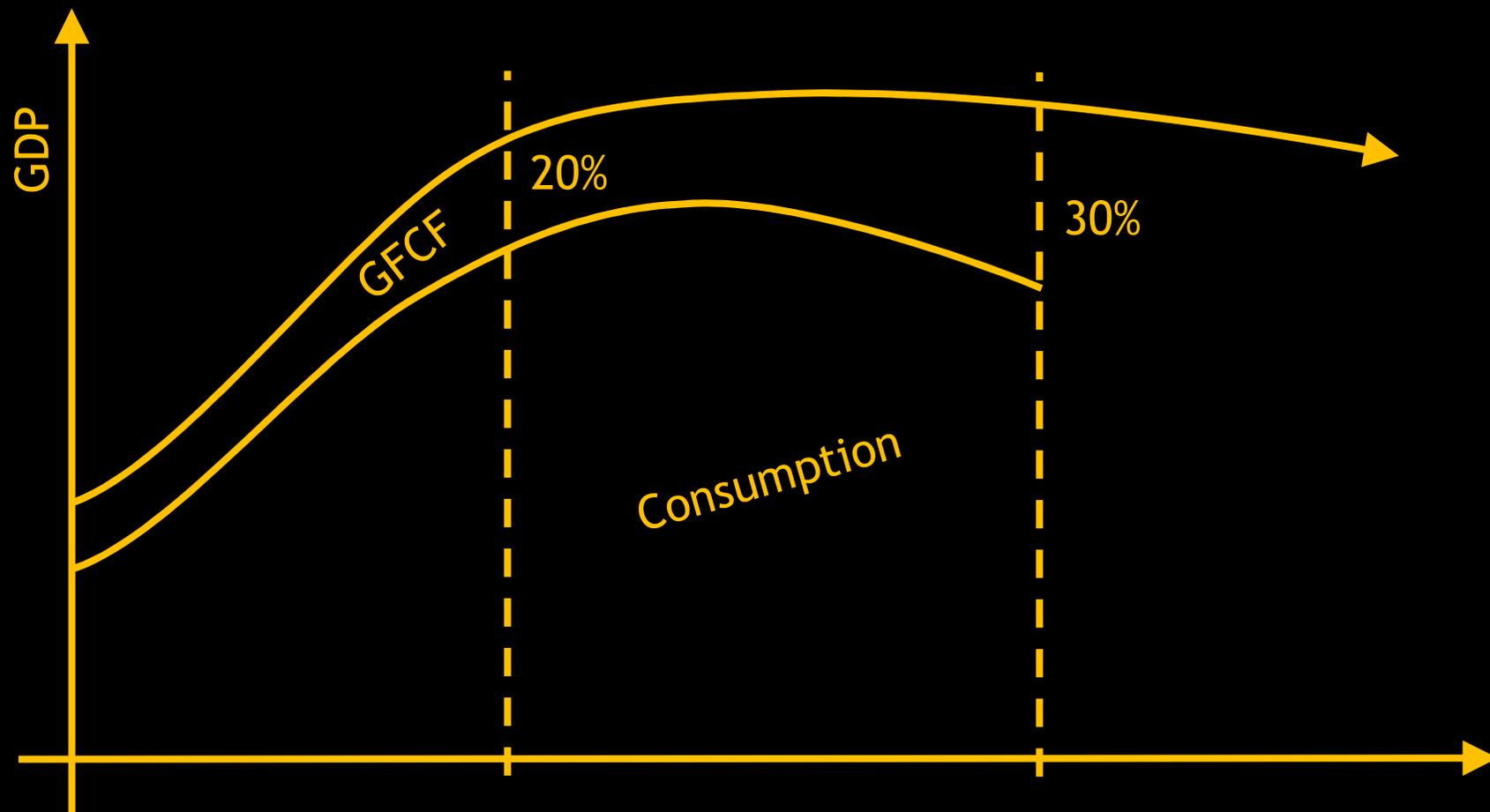
Elasticity



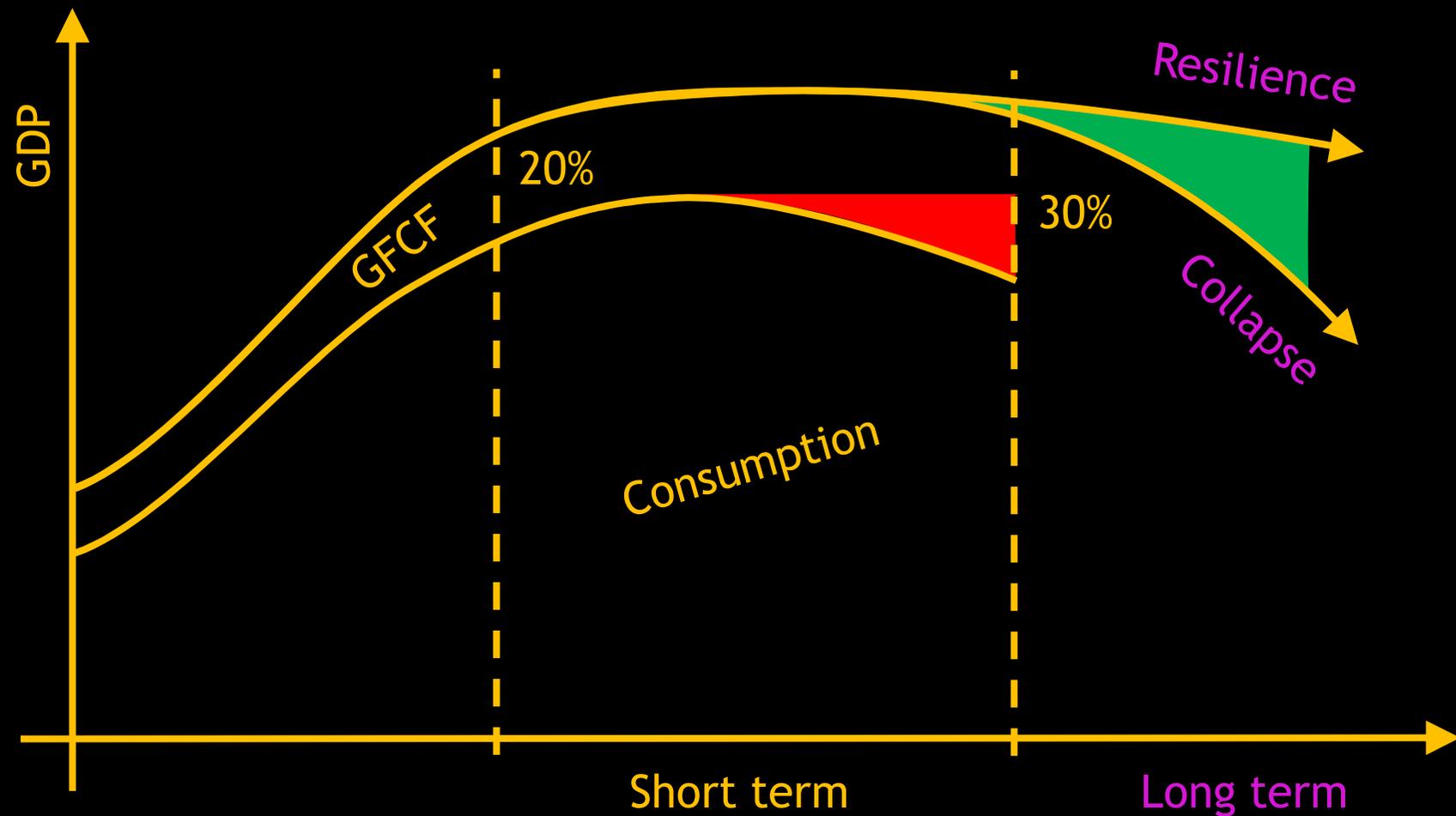
# Rethink wealth distribution



## 6) Short / long term issue



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“We have to solve climate and energy issues before the second half of the century”

Jean-Noël Giraud  
(quoted by Patrick Criqui)

Decoupling will be a big challenge!

