Discounting in economic analysis

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- Discounting is a central tool in inter-temporal economic analysis.
- The difficulty is that we have to compare current costs/benefits with future (long-run and uncertain) costs/benefits.
- The question of how to price the future is of course crucial for energy policy, and policies dealing with the climate change issue.
- Example: the social cost of carbon.

- Recent debates on the economics of climate change were triggered by the Stern review (2006). Prompted strong action to prevent climate change.
- There was a controversy with Nordhaus (JEL 2007) who on the contrary recommended a gradual response in terms of reducing emissions.
- One of the main difference turned out to be the value of the discount rate.
  They interpreted the parameters of the discount rate in very different ways.

- The term 'discounting' is used for different things: there is some ambiguity about its definition.
- The value of the discount rate is much debated. In the meantime, results of economic analyses are often very sensitive to this parameter.
- For long-run issues: deep uncertainty. This also affects the value of the discount rate, and may even call usual practices into question (Weitzman REStat 2009).

1. Definition and use of a discount rate.
2. Controversies around the value of the social discount rate (SDR).
3. Risk, uncertainty and equity: additional issues surrounding the SDR.

- A key tool of evaluate policies/projects is to compute their Present Value (PV).

- If the policy/project P generates (net) benefits b_t, its PV is:

$$PV(P) = \sum_{t=0}^{T} e^{-\rho t} b_t$$

  where ρ is the discount rate: we put less weight on future costs/benefits.

- If the PV is positive, the project/policy may be implemented.

- We can distinguish two approaches to discounting (Arrow et al. 2012): the *investment approach* and the *consumption approach*.

- The investment approach: if the rate of return on investments is positive, we need less than a euro today to generate a dollar tomorrow. Discounting is used as a tool to rank projects, or to take into account that we need to borrow money to make the investment.

- The consumption approach: it is socially more valuable to increase consumption today than to increase consumption tomorrow. Reasons: future generations are richer, their existence is uncertain, society has a 'pure' preference for earlier consumption.

- The two approaches may converge under very specific circumstances:
  - Markets are perfect and complete.
  - A 'representative agent' approach is appropriate to determine society's 'preferences'.

- The social cost of carbon (SCC) is the PV of damages from emitting one additional ton of carbon dioxide.
- Let d_t be the damage in period t. The SCC is:

$$\sum_{t=0}^{T} e^{-\rho t} d_t$$

- The value is very sensitive to the choice of a discount rate.

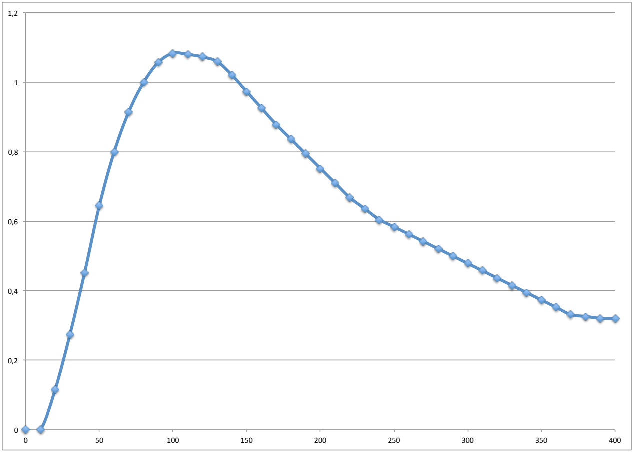

Figure : Damages induces by the emission of one ton of carbon in US$ 2000 using the DICE model (source: Nordhaus 2007)

- Aggregate the damages displayed in the figure using different discount rate yield very different SCC.

Discount rate (ρ)	$0,01\%$	1%	2%	4%	7%
SCC (US$ 2013)	4129	84	29	7	2

- Recommendation of values for the social cost of carbon for public policies have been made in several countries:
  - In the UK, the recommended carbon tax in 2010 was 27.30£= 30.5€ (DEFRA).
  - In France, the recommended carbon tax in 2010 was 32€ (Rapport Quinet).
  - In the US, the recommended carbon tax in 2010 was 21$=15€ (US Department of Energy).

- The carbon tax can be conceived as a Pigouvian tax used to have agents internalizing the cost of the externality they exert.
  If the SCC is the cost of the climate externality, the carbon tax should be equal to the social cost of carbon.

- More generally, abatement policies can assessed by looking at whether the PV of their costs is greater or less than the SCC (which is their benefit for each ton of carbon dioxide avoided).

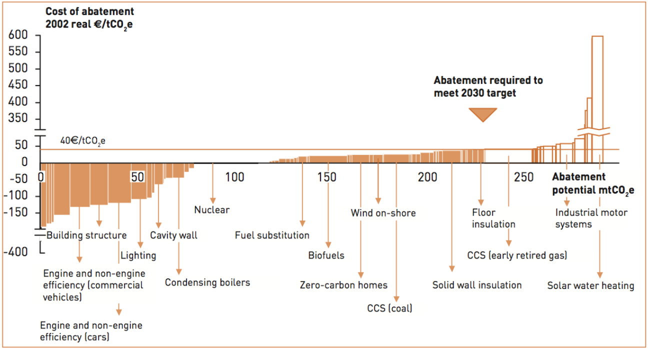

Figure : Abatement cost in the UK (source: McKinsey)

- There are two approaches to discounting: an investment based approach and a consumption-based approach.
- The two approaches converge only in very specific circumstances.
- Evaluation of policies/projects having very long-run impacts highly depends on the value of the discount rate.
- In the remainder, I will focus on the consumption-based approach to discute the value of a social discount rate (SDR).

- A common practice in economic theory consists in grounding cost-benefit analyses in a social welfare approach (see e.g. Drèze and Stern, 1987).

- Suppose that the society evaluates consumption paths using a 'social welfare function' (SWF)

$$\sum_{t=0}^{+\infty} \beta^t u(c_t). \tag{1}$$

- Consider a project yielding a small perturbation dc_t around the path c_t. The project is worth implementing if

$$\sum_{t=0}^{+\infty} \beta^t u(c_t + dc_t) - \sum_{t=0}^{+\infty} \beta^t u(c_t) \geq 0$$

which depends on wether

$$\sum_{t=0}^{+\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} dc_t \geq 0$$

- Denote $\rho_t(c) = -\frac{1}{t} \ln\left(\beta \frac{u'(c_t)}{u'(c_0)}\right)$.
- The evaluation of the future cost of the policy, $\sum_{t=1}^{+\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} dc_t$, can be written
$$\sum_{t=1}^{+\infty} e^{-\rho_t(c)t} dc_t.$$
- $\rho_t(c)$ is the social discount rate at period t.
In general, it depends on t and on the path c.

- Remark that

$$\ln\left(\frac{u'(c_t)}{u'(c_0)}\right) = \ln\left(\frac{u'(c_0+c_t-c_0)}{u'(c_0)}\right) \approx \ln\left(1+\frac{u''(c_0)(c_t-c_0)}{u'(c_0)}\right) \approx \frac{u''(c_0)c_0}{u'(c_0)}\cdot\frac{c_t-c_0}{c_0}$$

- Denoting $\delta = -\ln(\beta)$, $\eta(c_0) = -\frac{u''(c_0)c_0}{u'(c_0)}$ and $g_t(c) = \frac{c_t-c_0}{c_0}$, we obtain the approximation

$$\rho_t(c) \approx \delta + \eta(c_0)g_t(c)$$

- The key equation is then the 'Ramsey equation':

$$\rho_t(c) \approx \delta + \eta(c_0)g_t(c) \tag{2}$$

- How to interpret the 'parameters' in the above equations?
  - $\delta =$ 'pure time' discounting.
  - $\eta(c) = 1/\sigma(c)$, where σ is the inter-temporal elasticity of substitution. Also interpreted as 'risk aversion' and 'inequality aversion' (see the function $u(c) = c^{1-\eta}/(1-\eta)$).
  - $g_t(c)$ is the growth rate of consumption.

- Large controversy in the economic profession about the value of the SDR: Stern vs. Nordhaus.
- Both Stern and Nordhaus start from the social welfare function (1) and the Ramsey equation (2), but they have different interpretations yielding different values of the parameters in Eq. (2).
- Two approaches:
  - The 'ethical' (normative) approach (Stern): the social welfare should reflect normative views about the inter-temporal distribution of utility (Utilitarian approach).
  - The 'positive' approach (Nordhaus): the social welfare function represents the preferences of a representative agent.

- The debate (first) focused on the 'pure time' discounting parameter δ.
  - Stern views δ as a violation of intergenerational equity (following a long Utilitarian tradition: Sidgwick, Ramsey, etc.)
  - Nordhaus argued that we should 'respect' people preferences as reflected in market rates.

- Problems with Stern's position:
  - Why use the specific Utilitarian criterion?
  - Choice and interpretation of η are not clear.

- Problems with Nordhaus's position:
  - Which market rates? Are market working well in the case of very-long run investments.
  - A representative agent is a very specific aggregation of preferences. In particular, the preferences/interests of future persons are likely to be under-represented.

- The SDR has normative contend and we should be clear about it.
- The representative agent argument seems less convincing for issues involving a very long time horizon such as climate change.
- We should find new ways to elicit people views about such long-term problems, and make sure that future people interests are appropriately represented/taken into account.
- We may want to test the implications of alternative normative views.

Additional issues for the SDR:

- Risk/uncertainty: most discussed in the literature.
- Equity: intra-generational equity.
- Other issues (not much discussed): non-marginal policies, non-consumption impacts, risk on population size, other welfare criteria.

- The future evolution of whole economy is plagued with deep uncertainties.
- These uncertainties affect the valuation of future costs, hence the social discount rate and the SCC.
- Two kind of uncertainties have mainly been put forward (Gollier, 2012):
  - The uncertainty about the future level of consumption.
  - The uncertainty about the returns of the investments.

Standard approach to the risk on future consumption (Gollier 2008, 2012):

- Two-periods model, the social welfare function is:

$$W(c) = u(c_0) + e^{-\delta} Eu(\tilde{c}_1)$$

  with $u(c) = c^{1-\eta}/(1-\eta)$. Variable $\tilde{X} = \ln \tilde{c}_1 - \ln c_0$ is assumed to be normally distributed with mean μ and variance σ^2.

- The discount rate ρ can be defined as before:

$$\rho = -\ln\left(\frac{e^{-\delta} E\tilde{c}_1^{-\eta}}{c_0^{-\eta}}\right) = \delta - \ln\left(Ee^{-\eta\tilde{X}}\right)$$

- In this specific case, $Ee^{-\eta\tilde{X}} = e^{-\eta\left(E\tilde{X} - 0.5\eta Var(\tilde{X})\right)}$ so that:

$$\rho = \delta + \eta\mu - 0.5\ \eta^2\sigma^2.$$

- Weitzman (2009) uses the same model as Gollier, except that the variable $\tilde{X} = \ln \tilde{c}_1 - \ln c_0$ is normally distributed *but with an uncertain variance*.

- Using bayesian updating to infer the law of variable $\tilde{X}$, he shows that the growth rate has a fat-tail distribution (Student-t law). Weitzman shows that the SDR using the standard formula implies

$$\rho = -\infty$$

- Weitzman's result has however been criticized because it critically hinges on consumption being as close to zero as one wishes and marginal utility being infinite at zero.

- Another limitation of the Gollier result is that it assumes and i.i.d. growth process (in its more elaborated multi-period version).
- Recently, several economists suggested that we use a *declining* discount rate (DDR), based on the idea that there are serial correlations in the growth process (Arrow et al. 2013).
- The argument is coherent with existing practices in the UK (Green book 2003) and in France (Lebègue report 2005).
- Another reason: long run growth may be lower than short term growth.

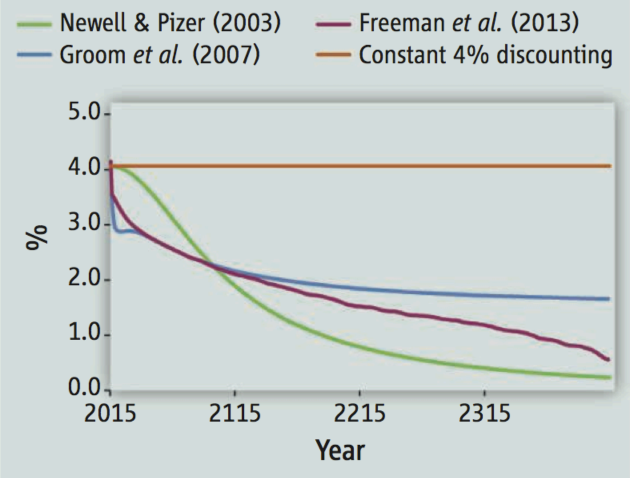

Figure : Term structure of the discount rate (source: Arrow et al. 2013)

- Another risk is the one on returns.
  We can adapt the usual discounting formula to obtain a discount rate for expected future costs and benefits (Gollier 2012; Fleurbaey and Zuber, 2014).

- Assume for instance a power utility and that the distribution of growth and returns is jointly normal, with μ_c the average consumption growth rate, σ_c^2 and σ_r^2 the variances of growth rate and of returns, and r their correlation.

- The discount rate is:

$$\rho = \delta + \eta\mu_c - 0.5\ \eta^2\sigma_c^2 + \eta r \sigma_c \sigma_r.$$

- More generally: more discounting when higher returns in 'better' states of the world.

- Inter-generational equity has been much discussed in the Stern vs. Nordhaus debate, the focus being the parameter δ (although the parameter η is also relevant from a distributive point of view).
- Intra-generational equity has been discussed a little bit in the literature on the SCC.
  People have resorted to welfare weights to weight costs and benefits of people with different wealth levels (Azar and Sterner, 1996; Fankhauser Tol and Pearce, 1997; Anthoff, Hepburn and Tol, 2009).

- Assume there are N country and that the social welfare function is:

$$\sum_{t=0}^{+\infty} e^{-\delta t} \sum_{i=1}^{N} n_{it} \frac{c_{it}^{1-\eta}}{1-\eta}$$

- The costs and benefits may not be shared equally among the different countries. Let α_{it} be the relative cost (or benefit) in country i and period t with respect to average cost (or benefit) $d\bar{c}_t$.

- The impact of this (marginal) policy is:

$$\sum_{t=0}^{+\infty} \sum_{i=1}^{N} e^{-\delta t} \alpha_{it} c_{it}^{-\eta} d\bar{c}_t$$

- One can redefine the social discount rate as

$$\rho_t = \delta - \frac{1}{t} \ln \left(\frac{\sum_{j=1}^{N} \alpha_{jt} c_{jt}^{-\eta}}{\sum_{i=1}^{N} \alpha_{i0} c_{i0}^{-\eta}} \right)$$

so that social welfare is increasing whenever:

$$\sum_{t=0}^{+\infty} e^{-\rho_t t} d\bar{c}_t > 0.$$

- Defining $\rho_t^{i,j} = \delta - \frac{1}{t} \ln \left(\frac{c_{jt}^{-\eta}}{c_{i0}^{-\eta}} \right)$, this may be rewritten:

$$\rho_t = \ln \left(\sum_{i=0}^{N} \alpha_{i0} \left(\sum_{j=1}^{N} \alpha_{jt} e^{-\rho_t^{i,j}} \right)^{-1} \right)$$

- We can also show that in the very long run ($t \to \infty$):

$$\rho_t \approx \max_i \min_j \rho_t^{i,j}$$

	USA	UE	Chine	Afrique	Average
USA	1.97	1.13	-1.67	-4.28	-2.44
UE	3.16	2.32	-0.48	-3.09	-1.25
Chine	7.30	6.46	3.66	1.05	2.89
Afrique	10.65	9.82	7.01	4.40	6.24

Table : Inter-regional discount rates in 2055, using RICE 2010, $\eta = 1.5$ and $\delta = 0$

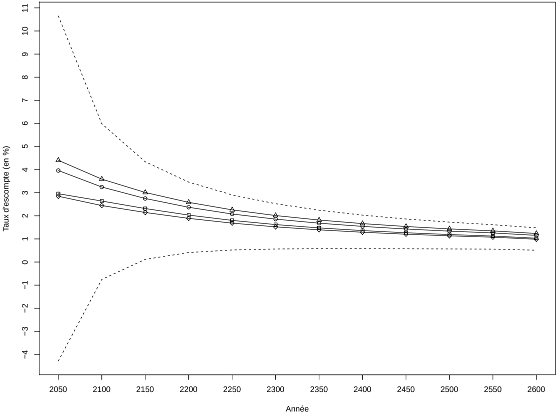

Figure : Social discount rate using different weights, using RICE 2010, $\eta = 1.5$ and $\delta = 0$

Here is a list of other issues that have not been much discussed:

- *Non-marginal impacts*. PV computations (and cost-benefit analysis in general) are appropriate when considering marginal changes to the economy. Large scale policies (climate policies for instance) may affect the whole growth process, as well as distribution and impacts.
  NB: that is why Nordhaus or Stern actually do not resort to PV calculations. But discounting is still interesting as a framing issue.
- *Non-consumption impacts*. Policies do not only affect consumption but also the probabilities of risky events or non consumption impacts (health, environmental goods).
  Most of them are not taken into account in economic assessments and require additional / alternative methods.

- *Risk on population / time horizon*. The risk on population size (the planning horizon) and the social valuation of population size are mostly unstudied.
  Example: there exist several versions of the 'Utilitarian' criterion used in the standard Ramsey equation: total vs average.

- *Alternative welfare criteria*. The Ramsey equation is derived from a very specific Utilitarian social welfare function. There is in principle no difficulty to apply the methodology to non-Utilitarian social welfare functions but little (almost nothing) is known about how the alternatives may change the discount rate.

- The computation of the social cost of carbon heavily relies on the underlying social evaluation criterion and the choice of the parameters.
- To decide on the parameters, we clearly need to choose the normative basis for the social discount rate.
- **The bottom-line: the social cost is not an 'ethic-free' number**.

- Risk and uncertainty may justify low discount rates for the far future. So does equity.
- The use of the discounted sum of damages is correct for marginal policies only. For large-scale policies, we need to rely directly on social criteria.
- The methodology applies only for certain consequences of policies, viz changes in income/consumption.
  Alternative methodologies must be developed to assess changes in risks and changes in the size of the population.

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